Time Series Lab 3

Apr 28, 2012

1. Simulate the following processes, try different sample sizes and parameter values. Let ε_t be i.i.d. $wn(0,1)$. You may try different distributions on ε (eg, Gaussian, Student-t, etc.).

(1) ARCH(1): $w_t = \sigma_t \varepsilon_t,\ \sigma_t^2 = c + a w_{t-1}^2$.

(2) GARCH(1,1): $w_t = \sigma_t \varepsilon_t,\ \sigma_t^2 = c + a w_{t-1}^2 + b\sigma_{t-1}^2$.

(3) AR(1)-GARCH(1,1): $X_t = \alpha X_{t-1} + w_t,\ w_t = \sigma_t \varepsilon_t,\ \sigma_t^2 = c + a w_{t-1}^2 + b\sigma_{t-1}^2$.

(4) ARMA(1,1)-GARCH(1,1): $X_t = \alpha X_{t-1} + w_t + \beta w_{t-1},\ w_t = \sigma_t \varepsilon_t,\ \sigma_t^2 = c + a w_{t-1}^2 + b\sigma_{t-1}^2$.

(5) GARCH(1,1)-M: $X_t = \beta \sigma_t + w_t,\ w_t = \sigma_t \varepsilon_t,\ \sigma_t^2 = c + a w_{t-1}^2 + b\sigma_{t-1}^2$.

2. Estimation of GARCH(1,1).

(1) Simulate the following GARCH(1,1) processes,

$$w_t = \sigma_t \varepsilon_t,\ \sigma_t^2 = 0.1 + 0.05 w_{t-1}^2 + 0.9\sigma_{t-1}^2,\ t = 1, 2, \ldots, n.$$

Let ε_t be i.i.d. $N(0,1)$.

(2) Estimate the parameters in the above model.

(3) Estimate and plot σ_t.

(4) Estimate ε_t. Conduct diagnostic tests on $\hat{\varepsilon}_t$.

3. Volatility of S&P 500 index return. (Data: Yahoo)

(1) Obtain daily data from Yahoo. Use the following command:

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SP500 = fetch(yahoo,'^GSPC','2010/01/01','2012/04/27','d').
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(2) Fit the daily return with an appropriate GARCH(p,q) model. Plot the estimated volatility.

(3) Fit the daily return with an appropriate GARCH-M model. Interpret your results.

(4) Fit the daily return with an appropriate EGARCH model. Interpret your results.