Monotonicity in Estimating Multiple Structural Breaks

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Abstract

We propose a monotonicity property for methods of estimating multiple structural breaks in linear regressions. A procedure with such a property yields a sequence of monotonically increasing sets of estimated break dates. Due to the uncertainty about the true number of breaks in finite samples, a monotone procedure offers a ranking of breaks from the least uncertain to the most. Most existing methods for estimating structural breaks do not enjoy monotonicity. We propose a new method that imposes monotonicity. Monte Carlo simulations show that the proposed procedure works well in finite samples. We also apply the procedure to a study of the structural changes in the Fed's monetary policy.

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1 Introduction

In applied studies involving time series regressions, researchers often segment the whole sample into several periods or regimes. Regressions with constant coefficients are then estimated in each of these regimes. In many cases, the dates of structural changes that separate adjacent regimes are treated as known. They may be major reforms, crises, or other extraordinary events. However, these events may not mark regime changes in data-generating processes (DGP) accurately. A break in DGP may happen long before the occurrence of major events due to the effect of expectation.

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It may also happen long after the major events (e.g., reforms) due to delays in implementation or manifestation of effects. Worse still, there may be structural breaks in the DGP that the researcher is unaware of.

Inaccurate or missing break dates can lead to incorrect segmentation and mistaken inferences. For applied researchers, it may thus be prudent to take an agnostic view on break dates and treat them as unknown. Not only are the break dates unknown, but the number of break dates is also unknown. Consequently, we may recast the problem of estimating multiple structural breaks into that of determining a sequence of sets of break dates. The sequence would be indexed by the number of breaks, $m = 1, 2, \ldots, \bar{m}$, where $\bar{m}$ is a predetermined upper bound of m. For each m, there would be a set of m break dates. Listing all of the sets in the sequence may reveal rich information about the time-varying nature of a model applied to real-world data.

And it would be desirable for the sequence to be monotonically increasing. This would allow estimated break dates to be ranked from the least uncertain to the most. We call methods that produce a monotonically increasing sequence of estimated break-date sets *monotone estimation*. Table 1 provides an example in which a sequence of three sets of estimated break dates (in columns) satisfies monotonicity. If there are breaks in the DGP, 1979Q3 would be the least uncertain among all candidates.

Table 1: Estimated Break Dates

m (# of breaks)	1	2	3
	1979Q3	1979Q3	1975Q1
		2001Q4	1979Q3
			2001Q4

The literature on estimating multiple structural breaks is well developed. Bai and Perron (1998) propose to estimate multiple structural breaks by least squares (LS). And Bai and Perron (2003) provide an efficient algorithm to calculate multiple breaks that globally minimizes the sum of squared residuals. Applying the idea of Lasso (Tibshirani, 1996), various authors (e.g., Bleakley and Vert (2011), Angelosante and Giannakis (2012)) propose to estimate multiple breaks in regressions by penalized least squares (PLS) with group-fused-Lasso (GFL) penalty. Qian and Su (2016) provide asymptotic theories of the resulting estimators of the break dates and regression coefficients.

However, most existing methods for estimating structural changes do not satisfy monotonicity. In this paper, we propose a new procedure that achieves monotonicity by adaptively penalizing

GFL. Monte Carlo simulations show that the cost of imposing monotonicity, in terms of the error in estimating the break dates when the number of breaks is known, is minimal.

The rest of the paper proceeds as follows. Section 2 defines the monotonicity property. Section 3 describes our new method. Section 4 presents Monte Carlo simulation results. Section 5 conducts an empirical study of the US monetary policy rule. Section 6 concludes.

2 Monotonicity of Structural-Break Estimation

Consider the following time series regression,

$$y_t = x_t'\beta_t + u_t,\ t = 1, ..., T, \tag{1}$$

where x_t is a vector of covariates, β_t is the corresponding vector of time-varying coefficients, and u_t is the error process. We assume that β_t is piecewise constant, containing m breaks ($m + 1$ regimes), with m being a small integer compared to T. Specifically, let T_j denote break dates for $j = 1, \dots, m$. We assume that $\beta_t = \alpha_j$ for $t = T_{j-1}, \dots, T_j - 1$ and $j = 1, \dots, m+1$. By convention, we have $T_0 = 1$ and $T_{m+1} = T + 1$.

Define $\theta_1 \equiv \beta_1$ and $\theta_t = \beta_t - \beta_{t-1}$ for $t > 1$. The set of structural breaks can be defined as $\mathcal{T} = \{t > 1 | \theta_t \neq 0\}$. Let $\hat{\theta}_t$ be the estimator for θ_t. The estimated set of structural breaks is thus $\hat{\mathcal{T}}_m = \{t > 1 | \hat{\theta}_t \neq 0\}$. We make the dependence of $\hat{\mathcal{T}}$ on m explicit. Note that m is generally unknown to applied researchers. Bai and Perron (1998) propose to determine m by testing the null hypothesis of m breaks against the alternative of $m - 1$ breaks. Qian and Su (2016) propose an information criterion to determine m. In this study, our objective is not to determine m. Instead, we estimate an m-indexed sequence of sets, each of which contains m possible break dates. And we define a desirable property applicable to both sequences of sets and procedures that yield sequences satisfying this property:

Definition: Monotonicity. Suppose that an estimation procedure yields a sequence of sets, $\hat{\mathcal{T}}_m$ for $m = 1, \dots, \bar{m}$. If $\hat{\mathcal{T}}_m \subset \hat{\mathcal{T}}_n$ whenever $m < n$, then we say that the estimation procedure enjoys monotonicity.

In other words, a monotone procedure guarantees a monotonically increasing set of estimated break dates. Some remarks are due regarding the monotonicity (or lack of) of existing approaches to the estimation of structural breaks. Let $\text{SSR}(\{T_1, \dots, T_m\}) = \sum_{j=1}^{m+1} \sum_{t=T_{j-1}}^{T_j - 1} (y_t - x_t'\alpha_j)^2$. For brevity, when minimizing SSR, we focus on selecting $\{T_j\}$ without mentioning $\{\alpha_j\}$, which are chosen jointly with $\{T_j\}$.

Remark 1: Sequential LS. The following procedure, which is based on minimizing least squares, is monotone.

(1) Choose $T_1^{(1)}$ that minimizes $SSR\left(\{T_1^{(1)}\}\right)$;

(2) Given $T_1^{(1)}$, insert $T_2^{(2)}$ that minimizes $SSR\left(\{T_1^{(1)}, T_2^{(2)}\}\right)$;

(3) Repeat till we obtain $T_{\bar{m}}^{(\bar{m})}$.

The sequence of estimated sets of break dates is thus

$$\begin{aligned} m = 1: &\quad \hat{\mathcal{T}}_1 \equiv \{T_1^{(1)}\} \\ m = 2: &\quad \hat{\mathcal{T}}_2 \equiv \{T_1^{(1)}, T_2^{(2)}\} \\ \vdots &\quad \vdots \\ m = \bar{m}: &\quad \hat{\mathcal{T}}_{\bar{m}} \equiv \{T_1^{(1)}, \ldots, T_{\bar{m}}^{(\bar{m})}\}. \end{aligned}$$

Remark 2: Global LS. The Global LS procedure described in Bai and Perron (1998) is *not* monotone:

(1) For $m = 1$, choose $T_1^{(1)}$ that minimizes $SSR\left(\{T_1^{(1)}\}\right)$;

(2) For $m = 2$, choose $T_1^{(2)}$ and $T_2^{(2)}$ that minimize $SSR\left(\{T_1^{(2)}, T_2^{(2)}\}\right)$;

(3) Repeat till m reaches $\bar{m}$.

The sequence of estimated sets of break dates is thus

$$\begin{aligned} m = 1: &\quad \hat{\mathcal{T}}_1 \equiv \{T_1^{(1)}\} \\ m = 2: &\quad \hat{\mathcal{T}}_2 \equiv \{T_1^{(2)}, T_2^{(2)}\} \\ \vdots &\quad \vdots \\ m = \bar{m}: &\quad \hat{\mathcal{T}}_{\bar{m}} \equiv \{T_1^{(\bar{m})}, \ldots, T_{\bar{m}}^{(\bar{m})}\}. \end{aligned}$$

Note that $T_1^{(1)}$ is generally different from the elements in $\hat{\mathcal{T}}_2$.

Remark 3: Penalized LS with Group Fused Lasso (PLS-GFL). PLS-GFL studied in Qian and Su (2016) does not guarantee monotonicity. PLS-GFL estimates break dates by minimizing

$$\frac{1}{T}\sum_{t=1}^{T}(y_t - \beta_t' x_t)^2 + \lambda \sum_{t=2}^{T} \|\beta_t - \beta_{t-1}\|, \tag{2}$$

or

$$\frac{1}{T}\sum_{t=1}^{T}\left(y_t - x_t' \sum_{s=1}^{t} \theta_s\right)^2 + \lambda \sum_{t=2}^{T} \|\theta_t\|, \tag{3}$$

where $\|\cdot\|$ is the Euclidean norm and $\lambda > 0$ is a tuning parameter for the GFL penalty. It is well known that there exists a $\lambda^* > 0$ such that if $\lambda > \lambda^*$, then $\hat{\theta}_t = 0$ for all $t \geq 2$, which corresponds to the case of no break. If for some $\lambda < \lambda^*$, $\|\hat{\theta}_{T_1}\| \neq 0$, then there is a break at T_1. Further decreasing λ would produce more breaks. The estimation procedure is as follows:

(1) Start from a $\lambda^{(0)} > \lambda^*$, which ensures no break;

(2) Search for a smaller tuning parameter $\lambda^{(1)} < \lambda^{(0)}$, such that $\lambda^{(1)}$ produces one break at $T_1^{(1)}$;

(3) Search for a even smaller tuning parameters $\lambda^{(2)} < \lambda^{(1)}$ such that $\lambda^{(2)}$ produces two breaks at $T_1^{(2)}$ and $T_2^{(2)}$;

(4) Repeat till we have $\bar{m}$ breaks at $T_1^{(\bar{m})}, \ldots, T_{\bar{m}}^{(\bar{m})}$.

The sequence of estimated sets of break dates is thus

$$\begin{aligned}
m = 1: &\quad \hat{\mathcal{T}}_1 \equiv \{T_1^{(1)}\} \\
m = 2: &\quad \hat{\mathcal{T}}_2 \equiv \{T_1^{(2)}, T_2^{(2)}\} \\
\vdots &\quad \vdots \\
m = \bar{m}: &\quad \hat{\mathcal{T}}_{\bar{m}} \equiv \{T_1^{(\bar{m})}, \ldots, T_{\bar{m}}^{(\bar{m})}\}.
\end{aligned}$$

Theoretically, PLS-GLS should be monotone since, if the procedure produces a strictly positive $\|\hat{\theta}_{T_j}\|$, the declining tuning parameter in subsequent rounds tends to preserve the positiveness of $\|\hat{\theta}_{T_j}\|$. Hence, the preservation of the break date T_j in all subsequent rounds. In practice, however, there are two problems. First, the practical implementation of the minimization of (3), using either the block coordinate descent algorithm or a general-purpose convex programming package such as CVX, employs a stopping criterion that is economic in terms of computation cost but may fail to ensure monotonicity. Second, PLS-GFL tends to produce more than one break around the true break date. To get rid of spurious breaks, Harchaoui and Lévy Leduc (2010) propose an algorithm called rDP, which essentially applies a dynamic-programming algorithm similar to Bai and Perron (2003) to the estimated break dates in the first stage. As an alternative, we may filter out spurious breaks by computing:

$$f(t) = \sum_{j=2}^{T} K\left(\frac{j-t}{h}\right) \|\hat{\theta}_j\|, \; t = 2, \ldots, T,$$

where $K(\cdot)$ is a kernel function and h is bandwidth. Intuitively, if there are positive $\|\hat{\theta}_j\|$ clustering around t, then the curve of $f(\cdot)$ would exhibit a peak around t. The location of the peak is then a likely suspect for a break date. By retaining only the peaks, we may filter out the spurious breaks surrounding the true one. The bandwidth h regulates the degree to which the filtering is performed. A higher h achieves a higher degree of filtering. In practice, the ad hoc choice of $h = 1$ works well, and the results are not sensitive to the choice of h. Since this post-processing algorithm is based on kernel smoothing and finding peaks, we call it ksPeak.

Both rDP and ksPeak, however, lead to more incidences of non-monotone estimation. In the following, we present a new method that is based on PLS-GFL and, at the same time, guarantees monotonicity.

3 PLS-AGFL

We may estimate break dates by solving the following:

$$\min_{\{\theta_t\}} \frac{1}{T} \sum_{t=1}^{T} \left(y_t - x_t' \sum_{s=1}^{T} \theta_s \right)^2 + \lambda \sum_{t=2}^{T} w_t \|\theta_t\|, \tag{4}$$

where w_t is a weighting process. Note that if we let $w_t = 1$ for all t, then the above problem reduces to (3). Suppose we aim to find up to $\bar{m}$ breaks. Our procedure starts with setting $w_t = 1$ for all $t \geq 2$ and finding a $\lambda^{(0)} > \lambda^*$ that produces no structural changes. Going forward, we then find a smaller tuning parameter $\lambda^{(1)} < \lambda^{(0)}$ that produces exactly one break, say, at $T_1^{(1)}$ (that is, $\|\hat{\theta}_{T_1^{(1)}}\| \neq 0$). Now set $w_{T_1^{(1)}} = 0$ and find a tuning parameter $\lambda^{(2)}$ that produces exactly two breaks. One of the two breaks must occur at $T_1^{(1)}$ since the penalty on $\|\theta_{T_1^{(1)}}\|$ is zero. We then repeat the process till we find $\bar{m}$ breaks. The algorithm is summarized as follows:

(1) Start from setting $w_t = 1$ for all t and $\lambda^{(0)} > \lambda^*$, which ensures no breaks;

(2) Search for a smaller tuning parameter $\lambda^{(1)} < \lambda^{(0)}$, such that $\lambda^{(1)}$ produces one break at $T_1^{(1)}$.

Set $w_{T_1^{(1)}} = 0$;

(3) Search for a tuning parameters $\lambda^{(2)}$ that produces two breaks at $T_1^{(1)}$ and $T_2^{(2)}$. Set $w_{T_1^{(1)}} = w_{T_2^{(2)}} = 0$;

(4) Repeat till we have $\bar{m}$ breaks at $T_1^{(1)}, T_2^{(2)}, \ldots, T_{\bar{m}}^{(\bar{m})}$.

Since the weighting process depends on estimated results in the previous round, we call this procedure "PLS with adaptive GFL" or, simply, PLS-AGFL. For any $m \leq \bar{m}$, the estimated set of breaks is given by $\hat{\mathcal{T}}_m \equiv \{T_1^{(1)}, T_2^{(2)}, \ldots, T_m^{(m)}\}$. As m increases, the set $\hat{\mathcal{T}}_m$ is monotonically increasing. Since the number of breaks is fundamentally unknown in most studies, we may interpret that the break at $T_1^{(1)}$ is the least uncertain, $T_2^{(2)}$ is the second least uncertain, and so on. Thus PLS-AGFL offers a method to rank structural breaks in terms of certainty facing econometricians.

4 Monte Carlo Simulation

We conduct Monte Carlo experiments to compare the finite-sample performance of PLS-AGFL with Sequential LS, Global LS, PLS-GFL, and PLS-GFL with two methods of post-filtering (rDP and ksPeak), when the number of breaks is known. Among these procedures, only the first two enjoy monotonicity. The purpose of the simulations is to show that the PLS-AGFL obtains monotonicity without incurring much cost in terms of finite-sample performance. We generate data from $y_t = \beta_t + \beta_t x_t + u_t$, where $\beta_t = \alpha_1 I\{0 < t \leq T/4\} + \alpha_2 I\{T/4 < t \leq T/2\} + \alpha_3 I\{T/2 < t \leq 3T/4\} + \alpha_4 I\{3T/4 < t \leq T\}$ with $I\{\cdot\}$ being an indicator function. The coefficients $\alpha_1, \ldots, \alpha_4$ are selected from the fixed set of $\{0.25, 0.5, 0.75, 1\}$. There are three breaks with different jump sizes. The absolute value of the biggest jump is 0.75 and that of the second biggest jump is 0.5. We experiment with five data-generating processes (DGP) for x_t and u_t:

(1) $x_t \sim i.i.d.\ N(0,1)$, $u_t \sim i.i.d.\ N(0, \sigma_u^2)$.

(2) $x_t = 0.5x_{t-1} + e_t$, $e_t \sim i.i.d. N(0, 0.75)$, $u_t \sim i.i.d.\ N(0, \sigma_u^2)$.

(3) x_t same as in (2), $u_t = \sigma_u v_t, v_t = 0.5v_{t-1} + \epsilon_t, \epsilon_t \sim i.i.d.\ N(0, 0.75)$.

(4) x_t same as in (2), $u_t = \sigma_u v_t$ with $v_t = \epsilon_t + 0.5\epsilon_{t-1}$ and $\epsilon_t \sim i.i.d.\ N(0, 0.8)$.

(5) x_t same as in (2), $u_t = \sigma_u \sqrt{h_t}\epsilon_t, h_t = 0.05 + 0.05u_{t-1}^2 + 0.9h_{t-1}$, $\epsilon_t \sim i.i.d.\ N(0,1)$.

DGP (1) is the benchmark case where both x_t and u_t are i.i.d. Gaussian. DGP (2) and (3) introduce AR(1) dynamics to x_t and u_t. DGP (4) considers an AR(1) regressor and an MA(1) error. DGP (5) considers GARCH(1,1) error with an AR(1) regressor. The specifications ensure that both x and v have unit variances. σ_u regulates the noise level. The number of repetitions is 1000.

Table 2 reports HD/T, the average Hausdorff distance between the estimated and true sets of break dates standardized by the sample size. Not surprisingly, when the noise level is low ($\sigma_u = 0.25$), the Global LS has the best performance. PLS-AGFL attains the second-highest performance, notably outperforming Sequential LS, which also enjoys monotonicity. Note that GFL without post-processing performs poorly. Between the two GFL methods with postprocessing, ksPeak performs substantially better. At elevated noise levels ($\sigma_u = 1$), PLS-AGFL demonstrates superior performance, outperforming both GFL with post-processing and Global LS. This somewhat surprising result may be understood by noting that PLS-AGFL successively utilizes information obtained in the previous round.

In summary, we may conclude that PLS-AGFL, with monotonicity imposed, does not incur much cost in terms of finite-sample performance. On the contrary, the adaptive nature of our approach—enabled by treating multiple-break estimation as a sequence of break-date set estimations—could offer advantages, particularly in high-noise environments.

5 Structural Breaks in Monetary Policy Rule

As an illustration of the proposed method, we estimate the following empirical monetary policy rule of the US federal funds rate:

$$r_t - \pi_t = \beta_{0t} + \beta_{1t}\pi_t + \beta_{2t}y_t + u_t, \tag{5}$$

where the subscript t represents quarter, r_t denotes the effective federal funds rate, π_t the core PCE inflation rate, y_t the GDP gap. The regression equation is adapted from Taylor (1993), where $\beta_{0t}, \beta_{1t}, \beta_{2t}$ were assumed constants. In fact, (5) reduces to the original Taylor rule if we set $\beta_{0t} = 1$, $\beta_{1t} = \beta_{2t} = 0.5$. Here we allow the coefficients to have structural breaks, reflecting shifts in the policymaking of the US Federal Reserve. We use quarterly data from FRED covering 1960Q1 to 2024Q4.

Table 3 shows the estimated sets of break dates from $m = 1$ to 5. Part of the table is already shown in the Introduction as an illustration. For each m, we also calculate an information criterion

Table 2: Average error of break-date estimation (HD/T)

		LS		PLS			
DGP	σ_u	Sequential	Global	AGFL	GFL	rDP	ksPeak
T=60							
(1)	0.25	0.0385	0.0278	0.0303	0.1605	0.0858	0.0629
(2)	0.25	0.0559	0.0373	0.0464	0.1976	0.1217	0.0962
(3)	0.25	0.0641	0.0452	0.0571	0.2067	0.1355	0.1054
(4)	0.25	0.0641	0.0439	0.0534	0.2055	0.1321	0.1078
(5)	0.25	0.0540	0.0371	0.0449	0.2002	0.1284	0.0962
(1)	1	0.2836	0.2702	0.1496	0.2498	0.2201	0.2079
(2)	1	0.2873	0.2704	0.1603	0.2540	0.2205	0.2092
(3)	1	0.2587	0.2441	0.1758	0.2701	0.2264	0.2081
(4)	1	0.2731	0.2671	0.1802	0.2601	0.2251	0.2117
(5)	1	0.2967	0.2859	0.1671	0.2671	0.2283	0.2151
T=120							
(1)	0.25	0.0185	0.0139	0.0150	0.1456	0.0712	0.0460
(2)	0.25	0.0244	0.0160	0.0198	0.1751	0.1025	0.0777
(3)	0.25	0.0340	0.0217	0.0269	0.1917	0.1305	0.1048
(4)	0.25	0.0311	0.0201	0.0257	0.1884	0.1131	0.0914
(5)	0.25	0.0246	0.0160	0.0209	0.1750	0.1039	0.0806
(1)	1	0.2352	0.2057	0.1052	0.2500	0.2232	0.2110
(2)	1	0.2412	0.2118	0.1149	0.2565	0.2316	0.2195
(3)	1	0.2611	0.2510	0.1560	0.2769	0.2382	0.2238
(4)	1	0.2595	0.2327	0.1440	0.2711	0.2387	0.2291
(5)	1	0.2559	0.2256	0.1219	0.2571	0.2267	0.2158

(IC) defined in Qian and Su (2016). For comparison, we also calculate IC for the case of $m = 0$. It is clear that the results satisfy monotonicity. The least uncertain break, according to our procedure, happens in the third quarter of 1979. Note that Paul Volcker, the celebrated chairman of the Federal Reserve, took office in the second quarter of 1979. He was widely credited with ending the Great Inflation of the US in the 1970s and early 1980s. And according to our procedure, the second least uncertain break happens in the fourth quarter of 2001. This may be related to the pivot, under the leadership of Alan Greenspan, to easy monetary policy following the burst of the Dot-Com Bubble, the 9.11 attack, and various corporate scandals that undermined the economy and the financial market. We omit the discussion of other break dates for brevity. Indeed, if we choose a model using IC (Qian and Su, 2016), the minimal IC points to the case of two breaks ($m = 2$).

Table 4 shows the regression results in each regime when we choose $m = 2$. For comparison, we also show results for the case of $m = 0$, where no break exists and all coefficients are constant.

Table 3: Estimated Break Dates in the US Fed Interest Rate Policy

m (# of breaks)	0	1	2	3	4	5
		1979Q3	1979Q3	1975Q1	1975Q1	1975Q1
			2001Q4	1979Q3	1979Q3	1979Q3
				2001Q4	1981Q1	1981Q1
					2001Q4	1985Q1
						2001Q4
IC	1.8265	1.7941	1.1852	1.2817	1.3508	1.3467

We make the following observations. First, if all coefficients were constant, the Taylor rule gives a poor fit to the data. When we allow coefficients to have two breaks at 1979Q3 and 2001Q4, the goodness of fit (R^2) within each one of the three regimes improves substantially. The substantially lower IC value for $m = 2$ compared to $m = 0$ reassures us that the benefit of improving the fit outweighs the cost of the increasing complexity of the model. Second, it is evident that the policy shifts at 1979Q3 and 2001Q4 are mainly about the response to inflation. The response to the GDP gap changes marginally, but the response to inflation experiences changes of sign. Finally, the rate hikes by Jerome Powell since early 2022, although a drastic series of actions, do not appear to constitute a shift in the philosophy of policymaking.

Table 4: Regression Results in Each Regime

Regime	intercept	π_t	y_t	R^2
$m = 2$				
1960Q1-1979Q2	1.8671***	−0.1547**	0.4458***	0.4443
	(0.3309)	(0.0698)	(0.0680)	
1979Q3-2001Q3	1.9003***	0.5341***	0.3324***	0.3242
	(0.3084)	(0.0771)	(0.0860)	
2001Q4-2024Q4	2.3471***	−1.0212***	0.6637***	0.4483
	(0.4346)	(0.1695)	(0.0889)	
$m = 0$				
1960Q1-2024Q4	0.7004***	0.3109***	0.3688***	0.1635
	(0.2601)	(0.0675)	(0.0645)	

Notes: *** p<0.01, ** p<0.05, * p<0.1

6 Conclusion

Treating multiple-break estimation as a sequence of break-date set estimations, we propose a monotonicity property for break-date estimation. We propose an adaptive procedure that imposes monotonicity. This procedure would be particularly valuable when the number of breaks is highly uncertain and a ranking of break dates in terms of certainty is needed.

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