Sample Problems

1. True or False

Note: If you find the following statements true, you should briefly prove them. If you find them false, you should correct them.

(a) The sample average of estimated residuals from OLS is approximately zero.

(b) The following model suffers from perfect collinearity,

$$\log(income) = \beta_0 + \beta_1 education + \beta_2 male + \beta_3 female + u,$$

where $male = 1$ for male individuals and 0 for female individuals, and $female = 1$ for female individuals and 0 for male individuals.

(c) In a simple regression of y on x, the more variation in x, the less accurate is the slope estimator.

(d) Given the following probit model on the decision of buying a car,

$$p = F(\beta_0 + \beta_1 income + \beta_2 age),$$

where $F(\cdot)$ is the cumulative distribution function of standard normal distribution. The partial effect of *income* on the probability of buying a car is β_1.

2. Weighted Least Square

Consider the savings function,

$$sav_i = \beta_0 + \beta_1 inc_i + u_i, \; u_i = \sqrt{inc_i} \cdot \varepsilon_i, \tag{1}$$

where *sav* denotes "saving" and *inc* denotes "income", and where ε_i is iid with $\mathbb{E}\varepsilon_i = 0$ and $\text{var}(\varepsilon_i) = \sigma^2$. We assume ε is independent of *inc*.

(a) Show that $\mathbb{E}(u|inc) = 0$, so that the exogeneity assumption in CLR (Classical Linear Regression) Assumptions still holds.

(b) Show that $\text{Var}(u|inc) = \sigma^2 inc$. Which assumption in CLR assumptions is violated? And what consequences does it have for estimation and making inferences?

(c) Instead of running (1), we can run the following regression, which is called "weighted least square",

$$\frac{1}{\sqrt{inc_i}} sav_i = \beta_0 \frac{1}{\sqrt{inc_i}} + \beta_1 \frac{1}{\sqrt{inc_i}} inc_i + \varepsilon_i. \tag{2}$$

Now, does homoscedasticity hold for (2)?

(d) Provide a discussion that supports the assumption that the variance of savings increases with family income.

3. Inferences Consider the following regression,

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_1 x_3 + \beta_4 x_4 + \beta_5 x_5 + u.$$

Show how you would test the following hypotheses by running two regressions and computing an F-statistic. Show explicitly the regressions you would run and the degrees of freedom of each test statistic.

(a) $\beta_1 = 0$.

(b) $\beta_1 = 0$ and $\beta_4 = \beta_5$.

(c) $\beta_1 = 0$, $\beta_3 = 0$, and $\beta_4/\beta_5 = 2$.

4. The Simple Regression Consider the following simple regression,

$$y = \beta_0 + \beta_1 x + u,$$

which satisfies the classical linear regression assumptions.

(a) Show that $\beta_0 = \mathbb{E}y - \beta_1 \mathbb{E}x$ and $\beta_1 = \frac{\text{cov}(x,y)}{\text{var}(x)}$.

(b) Define SST (Total Sum of Squares), SSE (Explained Sum of Squares), and SSR (Residual Sum of Squares). Let the OLS estimated model be,

$$y = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{u} = \hat{y} + \hat{u},$$

where $\hat{\beta}_0$ and $\hat{\beta}_1$ are OLS estimates, $\hat{u}$ is estimated residual, and $\hat{y}$ is fitted value.

(c) Prove $SST = SSE + SSR$.

5. Empirical Problem: The Effect of Parent Education on Income We first estimate the following model,

$$\log(wage) = \beta_0 + \beta_1 edu + \beta_2 edu^2 + \beta_3 expr + u. \tag{3}$$

The EViews output is reported in Figure 1.

(a) Discuss the economic meaning of β_1.

(b) From the results in Fig. 1, how much increase in wage would be expected to get if one obtains one more year of education.

To see how parents' education affect children's income, we run another regression,

$$\log(wage) = \beta_0 + \beta_1 edu + \beta_2 expr + \beta_3 mothedu + \beta_4 fathedu + u, \tag{4}$$

where $mothedu$ is the mother's education and $fathedu$ is the father's education, which is measured by the number of years of schooling. The estimated model is reported in Fig. 2, and the covariance matrix for the estimators is given in Fig. 3.

(c) Test the statement that parents' education does NOT have any influence on children's income.

(d) Test the statement that fathers' education has the SAME influence on children's income as mothers' education.

Dependent Variable: LOG(WAGE)
Method: Least Squares
Date: 06/08/09 Time: 12:41
Sample: 1 1230
Included observations: 1230

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.760258	0.367499	-2.068736	0.0388
EDU	0.303465	0.050207	6.044235	0.0000
EDU^2	-0.006401	0.001826	-3.505894	0.0005
EXPR	0.031761	0.006750	4.705510	0.0000

R-squared	0.184599	Mean dependent var	2.413807
Adjusted R-squared	0.182604	S.D. dependent var	0.593715
S.E. of regression	0.536777	Akaike info criterion	1.596780
Sum squared resid	353.2473	Schwarz criterion	1.613413
Log likelihood	-978.0197	F-statistic	92.51840
Durbin-Watson stat	1.867559	Prob(F-statistic)	0.000000

Figure 1: Model 1.

Dependent Variable: LOG(WAGE)
Method: Least Squares
Date: 06/08/09 Time: 12:44
Sample: 1 1230
Included observations: 1230

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.879362	0.367754	-2.391166	0.0169
EDU	0.278142	0.050156	5.545512	0.0000
EDU^2	-0.005979	0.001815	-3.294175	0.0010
EXPR	0.033961	0.006719	5.054246	0.0000
MOTHEDU	0.008469	0.008635	0.980712	0.3269
FATHEDU	0.019956	0.006007	3.322108	0.0009

R-squared	0.197889	Mean dependent var	2.413807
Adjusted R-squared	0.194612	S.D. dependent var	0.593715
S.E. of regression	0.532820	Akaike info criterion	1.583599
Sum squared resid	347.4900	Schwarz criterion	1.608550
Log likelihood	-967.9137	F-statistic	60.39465
Durbin-Watson stat	1.868336	Prob(F-statistic)	0.000000

Figure 2: Model 2.

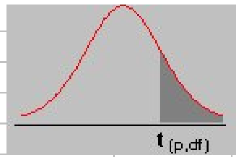
Coefficient Covariance Matrix						
	C	EDU	EDU^2	EXPR	MOTHEDU	FATHEDU
C	0.135243	-0.017026	0.000580	-0.001006	-0.000373	3.84E-05
EDU	-0.017026	0.002516	-8.94E-05	3.59E-05	-1.51E-05	-2.28E-05
EDU^2	0.000580	-8.94E-05	3.29E-06	1.27E-07	-7.64E-08	5.37E-07
EXPR	-0.001006	3.59E-05	1.27E-07	4.51E-05	-3.04E-07	2.75E-06
MOTHEDU	-0.000373	-1.51E-05	-7.64E-08	-3.04E-07	7.46E-05	-2.58E-05
FATHEDU	3.84E-05	-2.28E-05	5.37E-07	2.75E-06	-2.58E-05	3.61E-05

Figure 3: Model 2.

				5% Critical Values of the F Distribution						
				Numer DF						
Denom DF	1	2	3	4	5	6	7	8	9	10
10	4.97	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.10	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.97	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.56	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.33	3.47	3.07	2.84	2.69	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.26
25	4.24	3.39	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.17
40	4.09	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
inf	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

Figure 4: 5% Critical Values of F Distribution.

Right tail probability of t distribution



df\p	0.4	0.25	0.1	0.05	0.025	0.01	0.005	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	4.141
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.850
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.725
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.646
inf	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.291

Figure 5: Right-tail Probabilities of t Distribution