Asset Pricing Theory Problem Set 2

Due Date: next lecture

1. Let X and Y be martingales. Show

$$[X+Y]_t = [X]_t + [Y]_t + 2[X,Y]_t$$

2. Prove directly from the definition of Ito integral that

$$\int_0^t s dM_t = tM_t - \int_0^t M_s ds,$$

where M_t is a continuous martingale.

3. Verify that the following processes solve the given correspondent sde's.

(1) $X_t = e^{W_t}$ solves

$$dX_t = \frac{1}{2} X_t dt + X_t dW_t$$

(2) $X_t = W_t/(1+t)$ solves

$$dX_t = -\frac{1}{1+t} X_t dt + \frac{1}{1+t} dW_t.$$

4. Simulate the following diffusions:

(1) A Brownian motion W_t, $t \in [0,1]$.

(2) A geometric BM X_t that satisfies

$$X_0 = 1, \quad dX_t = 0.06 X_t dt + 0.02 X_t dW_t.$$

(3) A stationary Ornstein-Uhlenbeck process X_t that satisfies

$$dX_t = 0.5(0.03 - X_t)dt + 0.02 dW_t.$$

(Note: You should print at least one graph for each and show your Matlab or R code.)